

Optimization of CDA Cascade using Parameterization and Genetic Algorithm coupled with CFD

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ABSTRACT

Generating efficient blade profiles are the prerequisite for developing a high-performance compressor. Therefore, the goal of a blade design is to achieve the desired flow turning with minimum losses, within the constraint of the geometric orientation of the blade row. With the blade shapes becoming more sophisticated, the development of blade shapes is spanned in three generations: the first generation is the Circular-Arc Profile, the second generation is the Controlled Diffusion Airfoil (CDA), and the third generation is the optimized blades. In this present investigation a CDA profile has been selected and is parameterized with Bezier-PARSEC (BP) parameterization [1] method. From the BP parameters, the design variables are selected for the optimization using Genetic Algorithm (GA) as an optimization technique. The BP parameters include the aerodynamic and geometrical parameters. The objective function defined in the GA is to minimize the total pressure loss in the compressor cascade for a high subsonic inlet condition. The CFD software's Gambit and Fluent are used for simulation and total pressure loss is calculated in the cascade geometry. The coupling of BP parameterization, GA and CFD increases the convergence speed of the optimization. This investigation results an airfoil shape for the compressor cascade arrangement with optimum total pressure loss.

Keywords- CDA; Parameterization; Genetic Algorithm; Optimization; CFD.

1. INTRODUCTION

The Gas turbine engine manufacturers are looking for the efficient engines which gives higher thrust to weight ratio. This is possible in two ways, increase the maximum temperature in the combustion chamber and maximum pressure rise in compressor. Maximum temperature rise in the combustion chamber is limited to the turbine inlet temperature and turbine blade material. Hence to have a higher thrust to weight ratio pressure ratio should be higher in compressor. This can be achieved by running the compressor at higher speed, i.e. the flow becomes either high subsonic or transonic, but the sonic flow creates high losses in the cascade because of the formation of shock waves. Hence the other way to achieve an efficient compressor is by improving the compressor blade design. The current trend in compressors is to design an optimized blade with minimal pressure loss and higher pressure ratio. The present work carries out the optimization of the blade profile for the compressor cascade at high subsonic inlet flow conditions.

There have been a number of successful attempts to develop transonic airfoils and cascades. KHARAL and SALEEM [1]

in their work implemented artificial neural nets for their airfoil design. They described their airfoil geometry from the given cp-distribution using Bezier-PARSEC 3434 parameters [1] instead of using full profile coordinates. They concluded that the feed forward back propagation neural nets are more superior over the generalized regression and radial basis neural nets. ROGALSKY, DERKSEN and KOCABIYIK [2] compared different optimization techniques and found differential evaluation as the most efficient optimization technique as it produces a shape that closely modeled the targeted flow. The drawback of their work is the inability of the panel method to calculate the separated flow correctly. JAHANGIRIAN and SHAHROKHI [3] introduced a new method of parameterization and their new methods flexibility is investigated and applied to the reconstruction of the airfoil and the inverse design of the airfoil. They came to the conclusion that the convergence rate of the inverse design was remarkably increased when parameterization is used. OBAYASHI, TSUKAHARA and NAKAMURA [4] introduced a new multi objective method for cascade design. Using Multi Objective Genetic Algorithm [5] they produced physically reasonable solutions which perform better than the CDA in all the objectives such as high pressure rise, high turning angle and low total losses.

The present work includes, parameterization of CDA airfoil and its optimization for minimum total pressure loss. The optimization is carried out by the parameterization of CDA cascade by Bezier-PARSEC parameterization [1] and Genetic Algorithm [5] coupled with CFD.

2. PARAMETERIZATION

One of the critical points in the airfoil shape optimization and design is the geometrical parameterization. The parameterized airfoil should have a minimum number of design variables by considering both the geometrical and aerodynamic parameters for optimization. The number of design variables can be increased but it requires high computational time and enormous computational power.

2.1 BEZIER CURVES

The Bezier curves are named after their inventor Dr. Pierre Bezier, an engineer in the Renault car company. A Bezier curves of degree 'n' defined by 'n+1' control points of a polygon. The general expression for an nth order Bezier curve is,

$$P(u) = \sum_{i=0}^n P_i \left(\frac{n!}{i!(n-i)!} \right) u^i (1-u)^{n-i} \quad (1)$$

Where P_i is i^{th} control point. The parameter u goes from 0 to 1; with 0 at the zero-th control point and unity at the n^{th} control point. A Bezier parameterization is determined by its control points which are physical points in the plane. The number of design variables in Bezier curve is often so high that the computational time of the whole process becomes unaffordable.

A third order Bezier curve is given by:

$$X(u) = x_0(1-u)^3 + 3x_1u(1-u)^2 + 3x_2u^2(1-u) + x_3u^3 \quad (2)$$

$$Y(u) = y_0(1-u)^3 + 3y_1u(1-u)^2 + 3y_2u^2(1-u) + y_3u^3 \quad (3)$$

A fourth order Bezier curve is given by:

$$X(u) = x_0(1-u)^4 + 4x_1u(1-u)^3 + 6x_2u^2(1-u)^2 + 4x_3u^3(1-u) + x_4u^4 \quad (4)$$

$$Y(u) = y_0(1-u)^4 + 4y_1u(1-u)^3 + 6y_2u^2(1-u)^2 + 4y_3u^3(1-u) + y_4u^4 \quad (5)$$

2.2 PARSEC METHOD

Another common method for airfoil shape parameterization is PARSEC method which has been successfully applied to many airfoil design problems. This technique has been developed to control important aerodynamic features by using the finite number of design parameters. The basic eleven parameters that are used in PARSEC method including leading edge radius (r_{LE}), upper and lower crest locations (X_{UP} , Z_{UP} , X_{LO} , Z_{LO}) and curvatures (Z_{XXUP} , Z_{XXLO}), trailing edge coordinate (Z_{TE}) and direction (α_{TE}), trailing edge wedge angle (β_{TE}) and thickness (ΔZ_{TE}). PARSEC is one of the most

common and effective methods for airfoil representation in the design and optimization field. Despite its benefits over the Bezier curve, this method does not give the geometrical flexibility over the rear part of the airfoil. The studies of the effect of PARSEC in the aerodynamic inverse design of transonic airfoils show that despite its fast convergence, PARSEC is not able to converge to the desired airfoil shape.

2.3 BEZIER -PARSEC METHOD [1]

Due to the limitation of both Bezier and PARSEC parameterization methods, a new method has been adopted. Derksen and Rogalsky have introduced this method of parameterization by combining the advantages of both Bezier and PARSEC parameterizations called Bezier -PARSEC parameterization. In this method Bezier control points are determined in terms of the PARSEC parameters of an airfoil. In this method the airfoil is represented in terms of thickness and camber line, since these are directly related to the geometrical and aerodynamic performance of the airfoil.

In this case BP3434 scheme is used to parameterize the CDA. In BP3434, third order Bezier curves are used to describe the airfoil leading edge camber and thickness curves and fourth order curves describe the camber and thickness-trailing edge curves. Ten PARSEC parameters and five Bezier parameters are used to define the four Bezier curves. The PARSEC parameters are directly calculated from the airfoil geometry. The required BP3434 parameters are:

- PARSEC parameters are,
 - x_t , y_t , x_c , y_c , z_{te} , dz_{te} , α_{te} , β_{te} , γ_{le} , r_{le}
- Unknown Bezier control points are,
 - Camber curve b_0 , b_2 , b_{17}
 - Thickness curve b_8 , b_{15} , b_8 .

Table 1. Thickness profile control point coordinates

Leading edge		Trailing edge	
$x_0 = 0$	$y_0 = 0$	$x_0 = x_t$	$y_0 = y_t$
$x_1 = x_t$	$y_1 = b_8$	$x_1 = \frac{7x_c + \left(\frac{5b_8^2}{2r_{le}}\right)}{4}$	$y_1 = y_t$
$x_2 = 0$	$y_2 = y_c$	$x_2 = 3x_t + \frac{15b_8^2}{4r_{le}}$	$y_2 = \frac{(y_t + b_8)}{2}$
$x_3 = x_c$	$y_3 = y_c$	$x_3 = b_{15}$	$y_3 = dz_{te} + (1 - b_{15}) \tan(\beta_{te})$
		$x_4 = 1$	$y_4 = dz_{te}$

Table 2. Camber profile control point coordinates

Leading edge		Trailing edge	
$x_0 = 0$	$y_0 = 0$	$x_0 = x_c$	$y_0 = y_c$
$x_1 = b_0$	$y_1 = b_0 \tan(\gamma_{le})$	$x_1 = \frac{7x_c + \left(\frac{5b_8^2}{2r_{le}}\right)}{4}$	$y_1 = y_c$
$x_2 = b_2$	$y_2 = y_c$	$x_3 = \frac{13x_c - 8y_c \cot(\gamma_{le})}{6}$	$y_3 = \frac{(5y_c)}{6}$
$x_3 = x_c$	$y_3 = y_c$	$x_3 = b_{17}$	$y_2 = dz_{te} + (1 - b_{17}) \tan(\alpha_{te})$
		$x_4 = 1$	$y_4 = dz_{te}$

3. GENETIC ALGORITHM (GA) [5]

Presently Genetic Algorithm is a widely used optimization tool in aerospace field, because of its global search option and robustness. In each step the GA selects individuals in random from the current population, called parents and uses them to produce next generation individuals, called children. The generation of the children involved mainly three operations-

selection, crossover and mutation.

Selection- This selects the individuals and parents, for the next generation.

Crossover- It combines the two parents to form the children for the next generation.

Mutation- It applies random changes to individual parents to form children.

The difference of GA from the other optimization algorithms is that, the GA generates a population of points at each generation and the best point in the population approaches the optimal solution. GA has repeatedly modified population of individual solutions and over successive generations an optimal solution is reached.

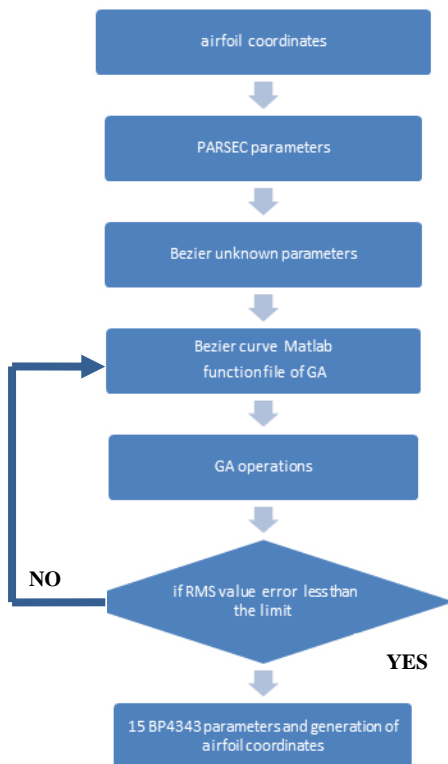


Fig. 1. BP3434 [1] parameterization algorithm

In the parameterization process (Fig-1) camber and thickness distribution are calculated from the base airfoil coordinates.

$$\begin{aligned} x_U &= x - y_t \sin \theta & y_u &= y_c + y_t \sin \theta \\ x_L &= x + y_t \sin \theta & y_u &= y_c - y_t \cos \theta \end{aligned} \quad (6)$$

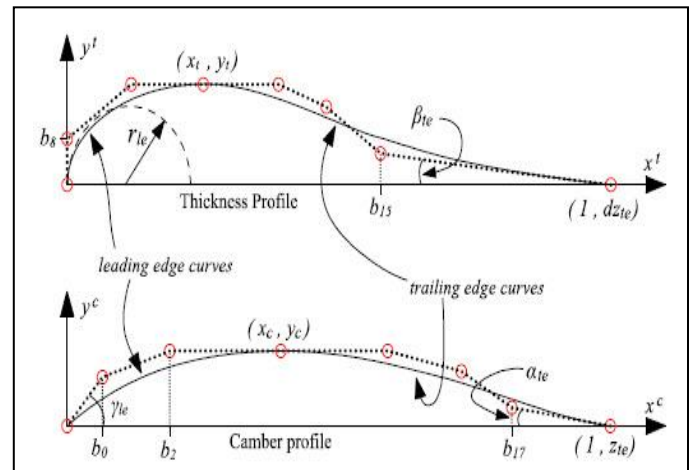


Fig. 2. Thickness and Camber distribution [1]

Camber and thickness distribution are divided into two sections, leading edge and trailing edge and a polynomial curve is formed with BP parameters for each section which is shown in Fig-2. GA minimizes the difference between the parameterized curve and base curve to find out the unknown Bezier parameters. After the root mean square value of error is reached the optimized BP parameters are obtained and the Fig-3 shows the parametrically generated airfoil and the base airfoil for the same geometry.

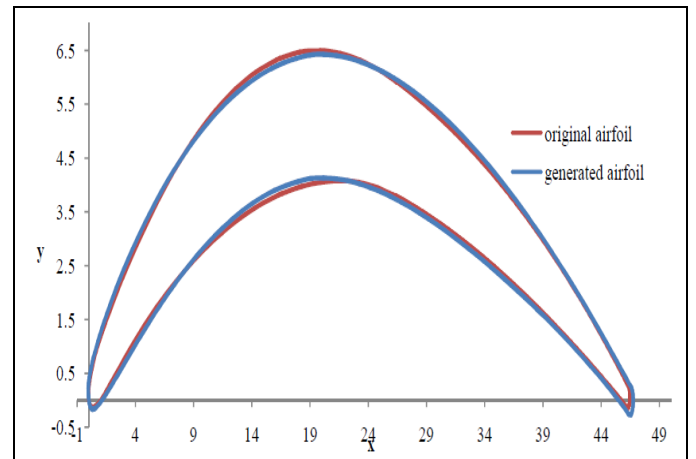


Fig. 3. Original and Parameterized Airfoil

4. OPTIMIZATION

The total pressure loss is the main reason for the increase in drag and loss in efficiency in the cascades at high speeds. The total pressure loss in the cascade is represented as [9],

$$\omega = \frac{\Delta p_0}{\left(\frac{1}{2} \rho c_x^2\right)} \quad (7)$$

$$\text{and } C_D = \omega \left(\frac{S}{l}\right) \cos^3 \alpha_m \quad (8)$$

and the efficiency of the compressor blade cascade is,

$$\eta_D = \left\{1 - \left[\frac{\Delta p_0}{\rho c_x^2 \tan \alpha_m (\tan \alpha_1 - \tan \alpha_2)}\right]\right\} \quad (9)$$

So by minimizing the total pressure loss the efficiency of the cascade can be increased and also drag can be minimized.

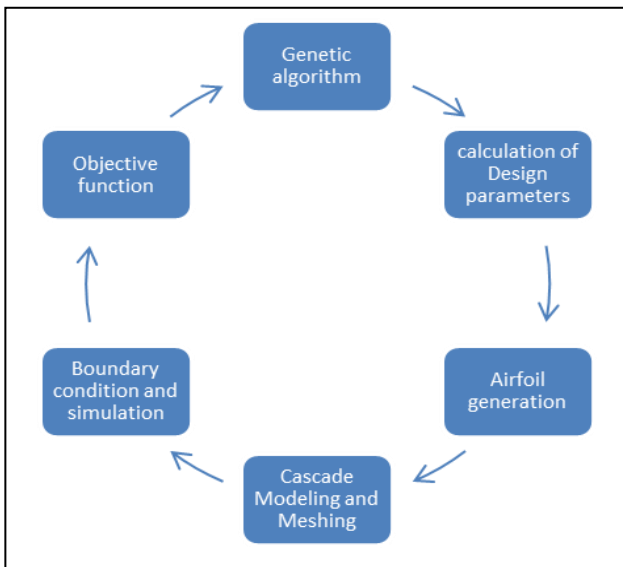


Fig. 4. Flow chart for the optimization

The objective of the present work is the optimization of the compressor cascade for high subsonic velocities. The optimization is meant for finding a profile section with minimal loss. In this investigation we selected a CDA, third stage of a compressor for the optimization. In the optimization process the modeling and meshing of the CDA are carried out in Gambit and the analysis is carried out in Fluent. The GA is called in the Matlab and coupled with airfoil generation and CFD software's (Batch mode). The Fig-4 shows the flow chart for the optimization process. For the optimization we first selected the design variables, which are obtained by the parameterization of airfoil section. The selected design variables are fifteen parameter of Bezier-PARSEC Parameterization. And secondly selected the Objective functions, the Objective function [5] for the optimization is defined as minimize total pressure loss Δp_0 . The selected design parameters are set as variables in the GA. In each iteration GA generates the children from the parents based on the genetic operations. The new airfoil section is generated from the children and as the next step the geometry modeling of the cascade section is carried out as and CFD analysis of the cascade is carried out to find the total pressure losses.

Table 3. Boundary conditions

Boundary	Boundary Condition
Inlet	Pressure inlet
Outlet	Pressure outlet
Airfoil	Wall
Outer boundaries	Periodic

For CFD analysis the mesh geometry is generated first in Gambit and then simulated in Fluent by applying the Boundary conditions as shown Table no-3. The applied Boundary conditions are: Inlet total pressure= 338000Pa, Inlet total temperature= 426K, Inlet flow angle= 44°, Inlet Mach no= 0.6, Outlet flow angle= 10°, which corresponds to a typical compressor stage under consideration. The parameters employed for CFD analysis include: density based solver with steady, second order upwind.

5. RESULTS AND DISCUSSION

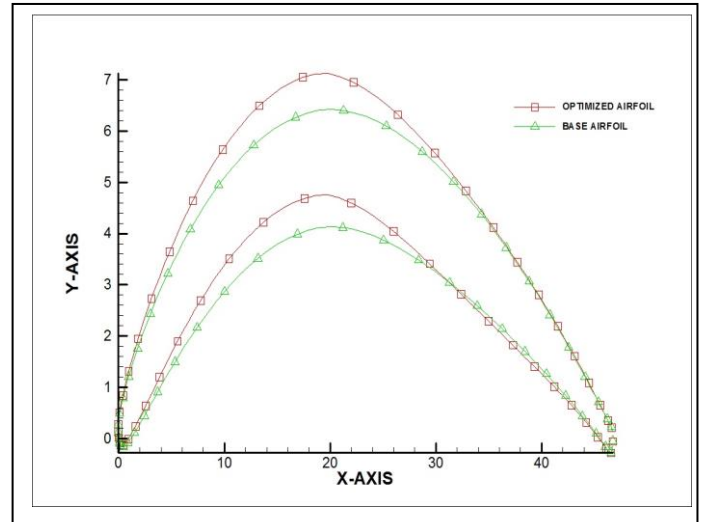


Fig. 5. Base and Optimized profiles

After a number of generations we obtained an optimized airfoil section. Fig-5 shows the comparison between the optimized and the base airfoil sections. From the optimized airfoil section it is concluded that the position of maximum thickness is kept towards the trailing edge and camber also increased a little. From the results of the simulation the objective function, i.e. total pressure loss, shows an optimized total pressure loss coefficient as shown in Table no-4.

Table 4. Results

Airfoil sections	Total pressure		Pressure loss coefficient
	Inlet	Outlet	
Base airfoil	338000	335150	0.0427
Optimized airfoil	338000	335370	0.0394

The Fig-6 shows the Mach no distribution over the optimized and base airfoil sections. The maximum Mach number is reduced from 0.81 to 0.76 in optimized airfoil. This reduction in Mach number also reduces total pressure losses in the cascade. Fig-7 represents the static pressure distribution over the optimized and base airfoils. A good distribution of static pressure is achieved in the optimized section as compared to base section; this may be because of the reduction in total pressure loss and also we obtained a smooth distribution of the Mach number.

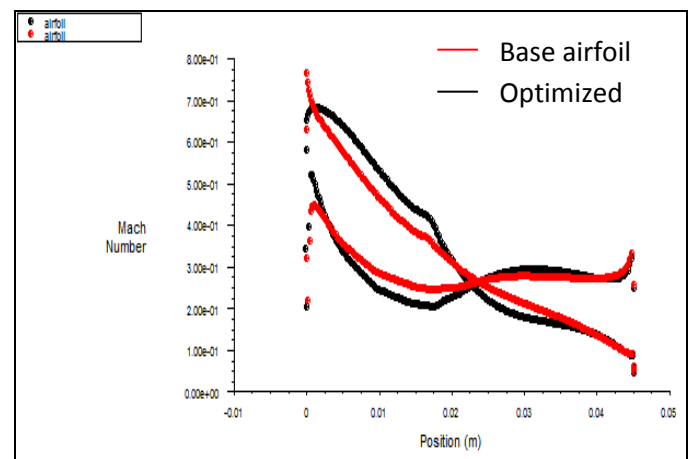


Fig. 6. Mach number distribution

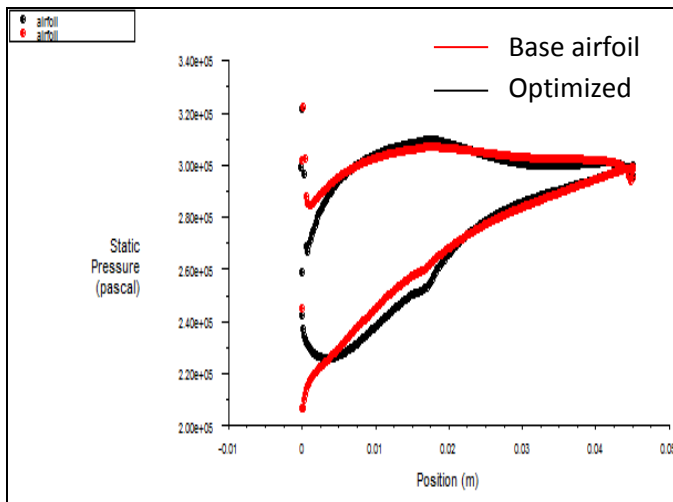


Fig. 7. Static pressure distribution

The Fig-6 shows the Mach no distribution over the optimized and base airfoil sections. The maximum Mach number is reduced from 0.8 to 0.7 in optimized airfoil. This reduction in Mach number also reduces total pressure losses in the cascade. Fig-7 represents the static pressure distribution over the optimized and base airfoils. A good distribution of static pressure has achieved in the optimized section as compared to base section; this may be because of a reduction in total pressure loss and also obtained a smooth distribution of the Mach number.



Fig. 8. Total pressure contours

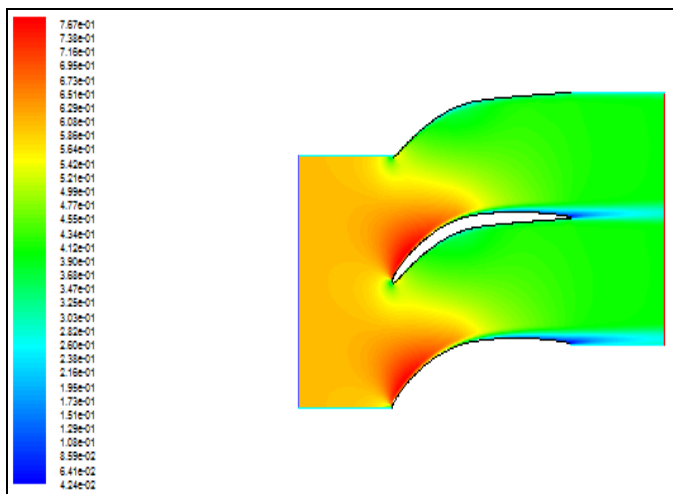


Fig. 9. Mach number contours

5. CONCLUSION

From the present investigation on optimization of compressor cascades with an inlet Mach number of 0.6, it is concluded that,

- Bezier-PARSEC Parameterization reduces the number of design variables for the optimization and design of the cascades. BP3434 couples the geometric and aerodynamic parameters of airfoil section which accelerates the optimization process and generates airfoil sections for high performance cascade.
- By using Genetic Algorithm a global optimum solution is achieved for the optimization of the CDA cascade within the design constraints for a large number of generations and minimize the total pressure loss coefficient for the typical cascade.
- It is observed that optimization time of the cascade is too high because of the CFD analysis of cascade uses a large amount of computational time for the flow analysis. Thus for reducing it we have to reduce the computational domain and use better CFD codes. Or use the minimum number of parameters which includes both geometric and aerodynamic parameters.
- Coupling of Bezier-PARSEC Parameterization with use of GA and CFD together, as presented herein, offers an optimal cascade profile. In the present case of optimization the total pressure loss coefficient reduces from 0.0427 to 0.0394. This means a reduction of the total pressure loss coefficient to a value of 7.72%.

6. ACKNOWLEDGEMENT:

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